CUET – 2023

Mathematics

Mock Paper – 1 (Solution)

Time: 45 min **Maximum Marks: 200**

General Instructions:

(i) Total duration of **Mathematics** Paper is **45** min.

(ii) You have to attempt 40 questions out of 50 in each Domain subjects.

(iii) All the questions provided are in MCQ format and have only single correct option.

(iv) Each question carries 5 marks. For each correct response, the candidate will get 5 marks. For each incorrect response, 1 mark will be deducted from the total score.

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1. If A = {1, 2, 3, 4} and B = {5, 6, 7, 8}, then the function which is one-one and onto is

(a) f_1= {(1, 5), (2, 7), (3, 8), (4, 6)}

(b) f_2 = {(1, 6), (2, 8), (3, 8), (4, 5)}

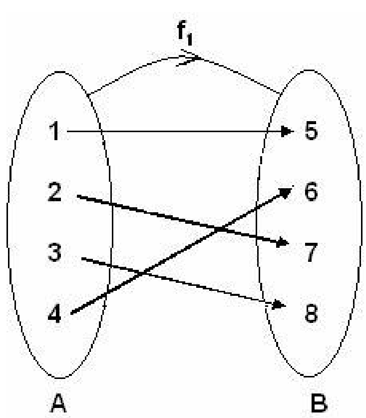
(c) f_3 = {(1, 5), (2, 7), (3, 8), (4, 5)}

(d) f_4 = {(1, 8), (2, 7), (3, 6), (4, 7)}

Correct answer – a: f_1= {(1, 5), (2, 7), (3, 8), (4, 6)}

Explanation:

The function f_1: A → B is given below:



In this function, the second entry in each ordered pair is unique and the set of second entries of the ordered pairs is the set B.

So, the function f_1 is one-one and onto.

2. The cosine function can be restricted to any interval of the type________, for its inverse to exist.
   (a) [nπ, (n+1) π]
   (b) [nπ/2, (n+1) π/2]
   (c) (nπ, (n+1) π)
   (d) (nπ/2, (n+1) π/2)

   Correct answer – c: (nπ, (n+1) π)

   Explanation:

   The cosine function restricted to any of the intervals [−π, 0], [0, π], [π, 2π] etc., is bijective with range [−1, 1].

   We can, therefore, define the inverse of cosine function in each of these intervals.

   The general form of these intervals is (nπ, (n+1) π).

3. A square matrix A is called an orthogonal matrix if
   (a) $AA^T = A^TA = I$
   (b) $AI = A$
   (c) $A^2 = I$
   (d) $A(A^T)^T = I$

   Correct answer – a: $AA^T = A^TA = I$

   Explanation:

   A square matrix A is called an orthogonal matrix when $AA^T = A^TA = I$

4. If $A = \begin{bmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{bmatrix}$ and $A\,(\text{adj } A) = k \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix}$, then the value of k is:
   (a) $\frac{1}{3}$
   (b) $\frac{1}{4}$
   (c) $\frac{1}{5}$
   (d) $\frac{1}{6}$

   Correct answer – c: $\frac{1}{5}$

   Explanation:

   We know that, $A\,(\text{adj } A) = |A|I$

   If $A = \begin{bmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{bmatrix}$, then

$|A| = \cos^2 x + \sin^2 x = 1$

$\therefore A\ (\text{adj } A) = I$

It is given that,

$A\ (\text{adj } A) = k\begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix} = 5k\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\Rightarrow A\ (\text{adj } A) = 5k\, I$

$\Rightarrow\ 5k = 1 \Rightarrow k = \dfrac{1}{5}$

5. If $A = \begin{bmatrix} 2 & -5 \\ -3 & 1 \end{bmatrix}$, then adj $(4A^2 + 9A)$ is equal to:

(a) $\begin{bmatrix} 94 & 105 \\ 63 & 73 \end{bmatrix}$

(b) $\begin{bmatrix} 73 & 63 \\ 105 & 94 \end{bmatrix}$

(c) $\begin{bmatrix} 105 & 73 \\ 63 & 94 \end{bmatrix}$

(d) $\begin{bmatrix} 73 & 105 \\ 63 & 94 \end{bmatrix}$

Correct answer – d: $\begin{bmatrix} \mathbf{73} & \mathbf{105} \\ \mathbf{63} & \mathbf{94} \end{bmatrix}$

Explanation:

Given: $A = \begin{bmatrix} 2 & -5 \\ -3 & 1 \end{bmatrix}$

$A^2 = \begin{bmatrix} 2 & -5 \\ -3 & 1 \end{bmatrix}\begin{bmatrix} 2 & -5 \\ -3 & 1 \end{bmatrix} = \begin{bmatrix} 4+15 & -10-5 \\ -6-3 & 15+1 \end{bmatrix} = \begin{bmatrix} 19 & -15 \\ -9 & 16 \end{bmatrix}$

$\therefore 4A^2 = 4\begin{bmatrix} 19 & -15 \\ -9 & 16 \end{bmatrix} = \begin{bmatrix} 76 & -60 \\ -36 & 64 \end{bmatrix}$

$9A = \begin{bmatrix} 18 & -45 \\ -27 & 9 \end{bmatrix}$

$4A^2 + 9A = \begin{bmatrix} 76 & -60 \\ -36 & 64 \end{bmatrix} + \begin{bmatrix} 18 & -45 \\ -27 & 9 \end{bmatrix} = \begin{bmatrix} 94 & -105 \\ -63 & 73 \end{bmatrix}$

$\text{adj}\ (4A^2 + 9A) = \begin{bmatrix} 73 & 63 \\ 105 & 94 \end{bmatrix}^T = \begin{bmatrix} 73 & 105 \\ 63 & 94 \end{bmatrix}$

6. If f: R → R and f(x) = x, then f o f =?

(a) R

(b) x

(c) 2x

(d) 3x

Correct answer – b: x

Explanation:

$(f \circ f)(x) = f\{f(x)\} = f\{x\} = x$

7. If $f(x) = |x - 2|$, then at $x = 2$, $f'(x)$ is
   (a) Continuous but not differentiable
   (b) Differentiable but not continuous
   (c) Continuous and differentiable both
   (d) Neither continuous nor differentiable

   Correct answer – a: Continuous but not differentiable

   Explanation:

   $f'(x) = -1$ at $x < 2$

   $f'(x) = 1$ at $x > 2$

   Therefore, not differentiable but continuous as it is a composition of two functions i.e., polynomial and modulus.

8. Find the second derivative of $e^x \cos x$.
   (a) $-e^x \sin x$
   (b) $-2e^x \cos x$
   (c) $-2e^x \sin x$
   (d) $e^x(\sin x + \cos x)$

   Correct answer – c: $-2e^x \sin x$

   Explanation:

   Let $y = e^x \cos x$

   $$\frac{dy}{dx} = e^x(\cos x - \sin x)$$

   $$\frac{d^2y}{dx^2} = e^x(\cos x - \sin x) + e^x(-\sin x - \cos x)$$

   $= -2e^x \sin x$

9. What is the maximum number of different elements needed to write a skew symmetric matrix of order n?
   (a) n^2
   (b) n
   (c) $n^2 - n$
   (d) $n^2 - n+1$

   Correct answer – d: $n^2 - n + 1$

   Explanation:

   To write a square matrix of order n, we need n^2 elements.

   In a skew symmetric matrix, all the diagonal elements are zeros.

   So, we need only $n^2 - n + 1$ elements to write a skew symmetric matrix of order n.

10. Evaluate: $\cos(\tan^{-1} x)$
    (a) $\frac{1}{1-x^2}$
    (b) $\frac{1}{1+x^2}$
    (c) $\frac{1}{\sqrt{1-x^2}}$
    (d) $\frac{1}{\sqrt{1+x^2}}$

    Correct answer – d: $\frac{1}{\sqrt{1+x^2}}$

    Explanation:

    We have,

    $$\cos(\tan^{-1} x) = \cos\left(\cos^{-1} \frac{1}{\sqrt{1+x^2}}\right) = \frac{1}{\sqrt{1+x^2}}$$

11. To check whether matrix B is an inverse of matrix A, we need to check
    (a) $AB^{-1} = I$
    (b) $BA^{-1} = I$
    (c) $AB = BA = I$
    (d) Either $AB = I$ or $BA = I$

    Correct answer – c: $AB = BA = I$

Explanation:

If A is a square matrix of order m, and if there exists another square matrix B of the same order m, such that AB = BA = I, then B is called the inverse matrix of A and it is denoted by A^{-1}.

In that case A is said to be invertible.

12. The greatest integer function is:

(a) continuous everywhere
(b) discontinuous everywhere
(c) continuous except at the integral values of x
(d) discontinuous except at end points

Correct answer – c: continuous except at the integral values of x

Explanation:

The graph of greatest integer function [x] breaks at integral values of x**.**

Thus, it is continuous everywhere except at the integral values of x.

13. A real function f is said to be continuous if it is continuous at every point in

(a) any interval of real numbers

(b) $[-\infty,\infty]$

(c) the range of f

(d) the domain of f

Correct answer – d: the domain of f

Explanation:

A real function f is said to be continuous if it is continuous at every point in the domain of f.

14. The area enclosed between the lines x = 2 and x = 7 is

(a) 7 units
(b) 5 units
(c) 2 units
(d) ∞

Correct answer – d: ∞

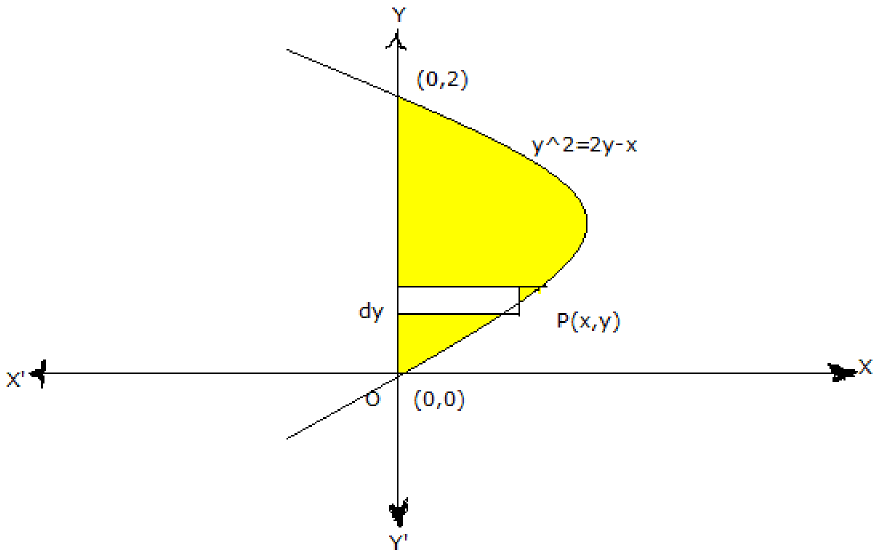
Explanation:

Area between two parallel lines is infinite.

15. Area of the region bounded by the curve $y^2 = 2y - x$ and y-axis is:
   (a) 3 sq. units
   (b) 4 sq. units
   (c) 3/4 sq. units
   (d) 4/3 sq. units

Correct answer – d: 4/3 sq. units

Explanation:



Putting x = 0 in $y^2 = 2y - x$, y = 0 and y = 2

$$Area = \int_0^2 x\,dy$$

$$\Rightarrow A = \int_0^2 (2y - y^2)\,dy$$

$$\Rightarrow A = \left[y^2 - \frac{y^3}{3} \right]_0^2 = 4 - \frac{8}{3} = \frac{4}{3}$$

Area = $\frac{4}{3}$ sq. units

16. The first order linear differential equation, where y is independent and x is dependent variable, is given by:

(a) $\frac{dy}{dx} + P(x)y = Q(x)$

(b) $\frac{dx}{dy} + P(x)y = Q(x)$

(c) $\frac{dx}{dy} + P(y)x = Q(x)$

(d) $\frac{dy}{dx} + P(y)x = Q(y)$

Correct answer – d: $\frac{dy}{dx} + P(y)x = Q(y)$

Explanation:

The first order linear differential equation, where y is independent and x is dependent variable, is:

$$\frac{dx}{dy} + P(y).x = Q(y)$$

Here, P(y) and Q(y) are the functions of y.

17. Which of the following is not a linear differential equation?

(a) $\frac{dy}{dx} + (1 + x^2)y = (1 + x^2)^2$

(b) $\frac{dy}{dx} + (1 + xy)y = x^2 + 2$

(c) $\frac{dy}{dx} + 1 + 2y = 4x$

(d) $\frac{dy}{dx} + (\frac{\cos x}{\sin x}) = \tan x$

Correct answer – b: $\frac{dy}{dx} + (1 + xy)y = x^2 + 2$

Explanation:

$\frac{dy}{dx} + (1 + xy)y = x^2 + 2$ is not a linear differential equation.

18. If O be the origin and $P_1(x_1, y_1, z_1)$ & $P_2(x_2, y_2, z_2)$ are two points, then the vector joining the points P_1 and P_2 is the vector P_1P_2 given by

(a) $OP_1 + OP_2$
(b) $OP_2 - OP_1$
(c) $OP_1.OP_2$
(d) $OP_1 - OP_2$

Correct answer – b: $OP_2 - OP_1$

Explanation:

$OP_1 + P_1P_2 = OP_2$ or $P_1P_2 = OP_2 – OP_1$

19. What is the additive identity of a vector?
    (a) zero vector
    (b) unit vector
    (c) The vector itself
    (d) Negative of the vector

    Correct answer – a: zero vector

    Explanation:

    $\vec{a}+\vec{0}=\vec{0}+\vec{a}=\vec{a}$

    Therefore, zero vector is the additive identity for a vector.

20. Area bounded by the curve y = cos x between x = 0 and x = 2π is
    (a) 0
    (b) 1 square unit
    (c) 2 square units
    (d) 4 square units

    Correct answer – d: 4 square units

    Explanation:

    The required area is given by

$$A=\int_0^{2\pi}\cos x dx$$

$$=\int_0^{\frac{\pi}{2}}\cos x dx+\int_{\frac{\pi}{2}}^{\frac{3\pi}{2}}\cos x dx+\int_{\frac{3\pi}{2}}^{2\pi}\cos x dx$$

$$=[\sin x]_0^{\frac{\pi}{2}}+[\sin x]_{\frac{\pi}{2}}^{\frac{3\pi}{2}}+[\sin x]_{\frac{3\pi}{2}}^{2\pi}$$

$$=1+2+1 \ldots (\text{Since area can't be} -\text{ve})$$

$$=4 \text{ sq. units}$$

21. The distance of the point (2, 3, –5) from the plane x + 2y – 2z = 9 is:
    (a) 2 units
    (b) 3/2 units
    (c) 3 units
    (d) 10/3 units

Correct answer – c: 3 units

Explanation:

The distance of the point (2, 3, – 5) from the plane x + 2y – 2z = 9 is:

$$d = \frac{2(1)+2(3)-2(-5)-9}{\sqrt{1+4+4}} = \frac{2+6+10-9}{\sqrt{9}} = \frac{9}{3} = 3$$

22. The Cartesian equation of the line passing through the points (a, b, c) and (a', b', c') is:

(a) $\frac{x-a}{a'-a} = \frac{y-b}{b'-b} = \frac{z-c}{c'-c}$

(b) $\frac{x-a}{a'+a} = \frac{y-b}{b'+b} = \frac{z-c}{c'+c}$

(c) $\frac{x+a}{a'-a} = \frac{y+b}{b'-b} = \frac{z+c}{c'-c}$

(d) $\frac{x+a}{a'+a} = \frac{y+b}{b'+b} = \frac{z+c}{c'+c}$

Correct answer – a: $\frac{x-a}{a'-a} = \frac{y-b}{b'-b} = \frac{z-c}{c'-c}$

Explanation:

The Cartesian equation of the line passing through the points (a, b, c) and (a', b', c') is:

$$\frac{x-a}{a'-a} = \frac{y-b}{b'-b} = \frac{z-c}{c'-c}$$

23. The length of the perpendicular from the origin to the plane 3x + 2y – 6z = 21 is:

(a) 7
(b) 14
(c) 3
(d) 21

Correct answer – c: 3

Explanation:

The perpendicular distance from the origin to the plane is

$$= \left|\frac{3(0)+2(0)-6(0)-21}{\sqrt{(3)^2+(2)^2+(-6)^2}}\right| = \left|\frac{-21}{\sqrt{49}}\right| = \frac{21}{7} = 3$$

24. Objective function of an LPP is
    (a) a constraint
    (b) a function to be optimized
    (c) a relation between the variables
    (d) equation in a line

    Correct answer – b: a function to be optimized

    Explanation:

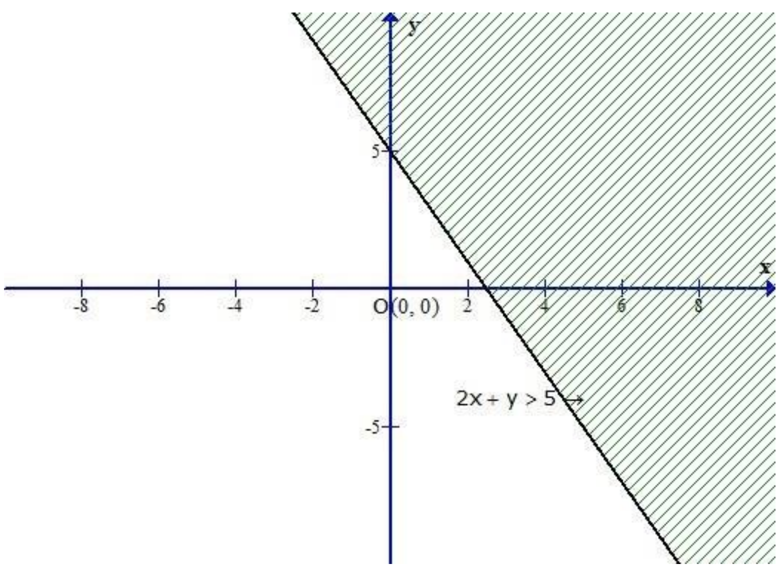
    Objective function of an LPP is a function to be optimized.

25. The solution set of the inequation 2x + y > 5 is
    (a) half plane that contains the origin
    (b) open half plane not containing the origin
    (c) whole xy-plane except the points lying on the line 2x + y = 5
    (d) points on line2x + y = 5

    Correct answer – b: open half plane not containing the origin

    Explanation:

    Consider the following graph.



    Thus, the graph of the given equation is the open half plane not containing the origin.

26. Let Z = ax + by be a linear objective function, then variables x and y are called ___________ variables.
    (a) dependent
    (b) independent
    (c) decision
    (d) continuous

Correct answer – c: Decision

Explanation:

If Z = ax + by is a linear objective function, then variables x and y are called decision variables.

27. Vectors that may subject to its parallel displacement without changing its magnitude and direction are called _________.

(a) free vectors
(b) co-initial vectors
(c) parallel vectors
(d) collinear vectors

Correct answer – a: Free vectors

Explanation:

Vectors that may subject to its parallel displacement without changing its magnitude and direction are called free vectors.

28. Two or more vectors having the same initial point are called

(a) unit vectors
(b) zero vectors
(c) co-initial vectors
(d) co-terminus vectors

Correct answer – c: co-initial vectors

Explanation:

Two or more vectors having the same initial point are called co-initial vectors.

29. The order and degree of the differential equation: $(y'')^2 + (y'')^3 + (y')^4 + y^5 = 0$ are respectively:

(a) 2 and 4
(b) 2 and 3
(c) 2 and 5
(d) 3 and 5

Correct answer – b: 2 and 3

Explanation:

Since, the highest differential coefficient of the equation $(y'')^2+(y'')^3+(y')^4+y^5=0$ is y'' and power of y'' is 3.

Therefore, order of the equation is 2 and degree is 3.

30. The solution of the below differential equation is:

$\frac{dy}{dx} = x \log x$

(a) $y = \frac{x^2}{2}\log x - \frac{x^2}{4} + c$

(b) $y = \frac{x^2}{2}\log x + \frac{x^2}{4} + c$

(c) $2y = x^2(\log x + 1) + c$

(d) $y = x^2(\log x + 1) + c$

Correct answer – a: $y = \frac{x^2}{2}\log x - \frac{x^2}{4} + c$

Explanation:

$$\frac{dy}{dx} = x\log x \Rightarrow dy = x\log x\ dx$$

$$\Rightarrow \int dy = \int x\log x\ dx$$

$$\Rightarrow y = \log x.\frac{x^2}{2} - \int \frac{1}{x}.\frac{x^2}{2}dx + c$$

$$\Rightarrow y = \frac{x^2}{2}\log x - \frac{x^2}{4} + c$$

31. The curves $x^2 + y^2 = 16$ and $y^2 = 6x$ intersects at

(a) $(2, 2\sqrt{3})$

(b) $(0, 2\sqrt{3})$

(c) $(2, 0)$

(d) $(0, 2)$

Correct answer – a: $(2, 2\sqrt{3})$

Explanation:

$x^2 + y^2 = 16; y^2 = 6x$

The points of intersection of the two curves $x^2 + y^2 = 16$ and $y^2 = 6x$

$x^2 + 6x = 16 \Rightarrow x^2 + 6x - 16 = 0 \Rightarrow (x+8)(x-2) = 0 \Rightarrow x = -8, 2$

But x is non negative $\Rightarrow x = 2 \Rightarrow y^2 = 6(2) = 12$

$\Rightarrow y = \sqrt{12} = 2\sqrt{3}$

$\therefore$ The points of intersection of the two curves is $(2, 2\sqrt{3})$

32. If $\int_0^{\alpha} \frac{1}{1+4x^2} dx = \frac{\pi}{8}$, the value of α is

(a) $\frac{1}{2}\tan\frac{\pi}{8}$

(b) $\frac{1}{2}$

(c) 1

(d) $\frac{1}{2\sqrt{2}}$

Correct answer – b: ½

Explanation:

Given: $\int_0^{\alpha} \frac{1}{1+4x^2} dx = \frac{\pi}{8}$

$\Rightarrow \frac{1}{2}\left[\tan^{-1}(2x)\right]_0^{\alpha} = \frac{\pi}{8}$

$\Rightarrow \tan^{-1}(2\alpha) = \frac{\pi}{4}$

$\Rightarrow 2\alpha = \tan\frac{\pi}{4} \Rightarrow \alpha = \frac{1}{2}$

33. The integration of the function log x is:

(a) x(log x – 1) + C
(b) (x log x – 1) + C
(c) x(log x + 1) + C
(d) (x log x + 1) + C

Correct answer – a: x(log x – 1) + C

Explanation:

$\int \log x \, dx = \int \log x.(1)\, dx = \log x.x - \int \frac{1}{x}.x\, dx$

$= x\log x - x + C = x(\log x - 1) + C$

34. The value of definite integral depends on

(a) the function and the interval
(b) the interval and the variable of integration
(c) the function and the variable of integration
(d) the function, the interval and the variable of integration

Correct answer – a: the function and the interval

Explanation:

Since in definite integrals, the variable of integration is a dummy variable. The value of definite integral depends only on the function and the interval not on the variable of integration.

35. Integration of sec x is:

(a) $\log|\sec x|+c$

(b) $\log|\tan x|+c$

(c) $\log|\sec x+\tan x|+c$

(d) $\frac{1}{\log(\sec x)}+c$

Correct answer – c: $\log|\sec x+\tan x|+c$

Explanation:

$Let \int \sec x dx = I = \int \frac{\sec x(\sec x+\tan x)}{(\sec x+\tan x)}dx$

$Put\ (\sec x+\tan x)=t \Rightarrow \left(\sec x\tan x+\sec^2 x\right)dx = dt$

$\Rightarrow \sec x\left(\sec x+\tan x\right)dx = dt$

$\therefore I = \int \frac{1}{t}dt = \log|t|+c = \log|\sec x+\tan x|+c$

36. Geometrically Rolle's theorem ensures that there is at least one point on the curve f(x), whose abscissa lies in (a, b) at which the tangent is

(a) parallel to the y-axis

(b) parallel to the x-axis

(c) parallel to the line y = x

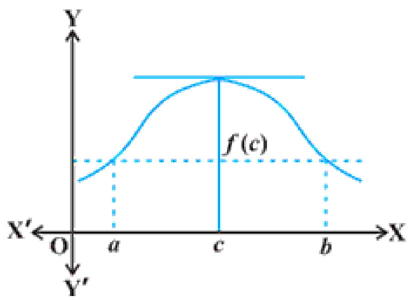
(d) parallel to the line joining the end points of the curve

Correct answer – b: parallel to the x-axis

Explanation:

Rolle's Theorem states that Let $f:[a,b]\to R$ be continuous on $[a,b]$ and be differentiable on (a,b), such that $f(a)=f(b)$ where a and b are some real numbers. Then there exists some c in (a,b) such that $f'(c)=0$.

The geometrical meaning of the statement is that when f(x) satisfies all relevant conditions , then ,there is atleast one point on the curve f(x) , whose absicca lies in (a, b) at which the tangent is parallel to the x axis



37. If order of the matrix A is 2 × 3, of matrix B is 3 × 2, and of matrix C is 3 × 3, then which one of the following is not defined?

(a) C (A + B')
(b) C (A + B')'
(c) BAC
(d) CB + A'

Correct answer – a: C (A + B')

Explanation:

A + B' is a matrix of order 2 × 3

Since, C is a matrix of order 3 × 3.

So, C (A + B') is not defined.

38. If A is a matrix of order m × n and B is a matrix of order l × p. The product AB of two matrices is defined if,

(a) n = p
(b) m = p
(c) n = l
(d) m = l

Correct answer – c: n = l

Explanation:

The product of two matrices A and B is defined if the number of columns of A is equal to the number of rows of B.

So here, AB is defined if $n = l$.

39. If $x + y + z = xyz$, then $\tan^{-1}x + \tan^{-1}y + \tan^{-1}z =$

 (a) π
 (b) $\pi/2$
 (c) 1
 (d) $\tan^{-1}(xyz)$

Correct answer – a: π

Explanation:

$$\tan^{-1} x + \tan^{-1} y + \tan^{-1} z = \tan^{-1}\left(\frac{x + y + z - xyz}{1 - xy - yz - zx}\right) = \tan^{-1} 0 = \pi$$

40. If $A = N \times N$ and * be any binary operation on A defined by $(a, b) * (c, d) = (a + c, b + d)$, then the binary operation is

 (a) commutative
 (b) associative
 (c) commutative and associative
 (d) commutative but not associative

Correct answer – c: commutative and associative

Explanation:

$(a, b) * (c, d) = (a + c, b + d)$ and $(c, d) * (a, b) = (c + a, d + b) = (a + c, b + d) = (a, b) * (c, d)$ So, * is commutative.

$(a, b) * [(c, d) * (e, f)] = (a, b) * (c + e, d + f) = (a + c + e, b + d + f)$

Also, $[(a, b) * (c, d)] * (e, f) = (a + c, b + d) * (e, f) = (a + c + e, b + d + f)$.

So, * is associative also.

Hence, * is commutative and associative.

41. What is the probability of $(A \cup R) \cap S$?

(a) $\frac{1}{6}$

(b) $\frac{2}{6}$

(c) $\frac{3}{6}$

(d) $\frac{1}{3}$

Correct Option – c: $\frac{3}{6}$

Explanation:

$(A \cup R) \cap S = \{1, 2, 5\}$ and Sample Space $= \{1, 2, 3, 4, 5, 6\}$

$$P((A \cup R) \cap S) = \frac{n((A \cup R) \cap S)}{n(\text{Sample space})} = \frac{3}{6}$$

42. The value of $P(A \mid R)$ is

(a) $\frac{1}{6}$

(b) $\frac{2}{6}$

(c) $\frac{3}{6}$

(d) $\frac{1}{3}$

Correct Option – d: $\frac{1}{3}$

Explanation:

Here, sample space $= \{1, 2, 3, 4, 5, 6\}$

$A \cap R = \{5\}$, $R \cap S = \{2, 5\}$, $A \cap S = \{1, 5\}$, $A \cap R \cap S = \{5\}$, $(A \cup R) \cap S = \{1, 2, 5\}$

$$P(A) = \frac{2}{6} = \frac{1}{3}, P(R) = \frac{3}{6} = \frac{1}{2}, P(S) = \frac{3}{6} = \frac{1}{2}$$

$$\text{Also, } P(A \cap R) = \frac{1}{6}, P(R \cap S) = \frac{2}{6}, P(A \cap S) = \frac{2}{6}$$

$$P(A \cap R \cap S) = \frac{1}{6} \text{ and } P((A \cup R) \cap S) = \frac{3}{6}$$

$$P(A|R) = \frac{P(A \cap R)}{P(R)} = \frac{\frac{1}{6}}{\frac{3}{6}} = \frac{1}{3}$$

43. Find the value of P(R | S).

(a) $\frac{2}{3}$

(b) $\frac{3}{6}$

(c) $\frac{1}{3}$

(d) $\frac{4}{3}$

Correct Option – a: $\frac{2}{3}$

Explanation:

$P(R \cap S) = \frac{2}{6}, P(R) = \frac{3}{6}$

$$P(R|S) = \frac{P(R \cap S)}{P(R)} = \frac{\frac{2}{\cancel{6}}}{\frac{3}{6}} = \frac{2}{3}$$

44. The values of P(A∩R | S) and P(A | S) are respectively

(a) $\frac{2}{3}$ and $\frac{1}{3}$

(b) $\frac{1}{3}$ and $\frac{2}{3}$

(c) $\frac{1}{6}$ and $\frac{3}{6}$

(d) $\frac{3}{6}$ and $\frac{1}{6}$

Correct Option – b: $\frac{1}{3}$ and $\frac{2}{3}$

Explanation:

$$P(A \cap R \mid S) = \frac{P(A \cap R \cap S)}{P(S)} = \frac{\frac{1}{\cancel{6}}}{\frac{3}{6}} = \frac{1}{3}$$

$$P(A|S) = \frac{P(A \cap S)}{P(S)} = \frac{\frac{2}{\cancel{6}}}{\frac{3}{6}} = \frac{2}{3}$$

45. Find the values of P(A∪R | S) and P(R∩S | A) respectively.

(a) $\frac{2}{5}$ and $\frac{1}{5}$

(b) $\frac{1}{5}$ and $\frac{2}{5}$

(c) 1 and $\frac{1}{2}$

(d) $\frac{1}{2}$ and 1

Correct Option – c: 1 and $\frac{1}{2}$

Explanation:

$$P(A \cup R \mid S) = \frac{P((A \cup R) \cap S)}{P(S)} = \frac{\frac{3}{\cancel{6}}}{\frac{3}{6}} = 1$$

$$P(R \cap S \mid A) = \frac{P(R \cap S \cap A)}{P(A)} = \frac{\frac{1}{\cancel{6}}}{\frac{2}{6}} = \frac{1}{2}$$

46. The revenue, R as a function of x can be expressed as

(a) $15x - \frac{x^2}{3000}$

(b) $15 - \frac{x^2}{3000}$

(c) $15x - \frac{1}{3000}$

(d) $15x - \frac{x}{3000}$

Correct option – a: $15x - \frac{x^2}{3000}$

Explanation:

The revenue function R(x) is given by

$$R(x) = p(x) \times x = \left(15 - \frac{x}{3000}\right)x$$

$$\Rightarrow R(x) = 15x - \frac{x^2}{3000}$$

47. The range of x is

(a) [0, 24000]

(b) [24000, 36000]

(c) [0, 36000]

(d) [12000, 24000]

Correct option – c: [0, 36000]

Explanation:

As the number of participants can be up to 36000.

So, the range of x is [0, 36000].

48. The value of x for which the revenue is maximum, is

(a) 20000

(b) 22500

(c) 21000

(d) 25000

Correct option – b: 22500

Explanation:

Since, $R(x) = 15x - \frac{x^2}{3000}$

$\Rightarrow R'(x) = 15 - \frac{x}{1500}$

For maxima/minima, R'(x) = 0

$\Rightarrow 15 - \frac{x}{1500} = 0$

$\Rightarrow x = 22500$

Again differentiating, we get

$R'(x) = -\frac{1}{1500} < 0$

At x = 22500, f''(x) < 0.

Hence, x = 22500 is the point of maxima.

49. If the revenue is maximum, the price of the ticket is

(a) Rs. 5.5

(b) Rs. 6

(c) Rs. 7.5

(d) Rs. 8

Correct option – c: Rs. 7.5

Explanation:

The revenue will be maximum at x = 22500

Therefore, price of a ticket is

$$15 - \frac{22500}{3000} = \text{Rs. } 7.5$$

50. How many students must participate so that the revenue is maximized?

(a) 21000

(b) 21500

(c) 22000

(d) 22500

Correct option – d: 22500

Explanation:

Number of students will be equal to the number of tickets sold.

Therefore, required number of students = 22500.